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Technical Notes

Blunted-Cone Heat Shields of Atmospheric Entry Vehicles

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Introduction

ATMOSPHERIC entry of a spacecraft generates extreme temperatures (>5000 K) due to its high speed and the resulting aerodynamic heating. Such extreme conditions require special design considerations to ensure safe passage of the craft through the atmosphere. One commonly employed technique is to use a heat shield. This device protects the craft from the high temperatures generated and also provides the necessary aerodynamic braking and stability for controlled entry through the atmosphere.

As the vehicle travels through the atmosphere, gas molecules interact with the heat shield, producing an aerodynamic force and torque. To ensure controlled entry, this torque must act to stabilize the craft should it stray from its required path. Vehicles entering the atmosphere travel through a wide array of gas densities, resulting in flows that range from free molecular to continuum in nature. It is important, then, that the heat shield shape produces stabilizing torques in all regimes. The material of the heat shield is sometimes chosen to be ablative, which reduces the heat flux at its surface [1]. However, ablation also changes the shape of the heat shield as the craft passes through the atmosphere and, hence, the dynamics of flow around it.

The shape of the heat shield used varies considerably between spacecraft, and spherical and blunted-cone geometries are often employed. The static stability of craft with spherical heat shields, at small angles of attack, is easily determined and requires the center of mass of the craft to be upstream from the center of curvature of the heat shield [1]. Sphere-cone geometries, which exhibit a conical section blunted at the tip, present a more formidable challenge to the calculation of their stability and are also widely used [2–5]. Here, we use the calculus of variations to show that the generic shape of the sphere-cone heat shield can be derived mathematically by requiring that the aerodynamic torque be insensitive to small shape changes. The resulting universal shape yields invariance in static stability due to minor heat shield damage and is maximally stable.

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Theoretical Model

We use Newtonian impact theory and free molecular (FM) theory [1]. Newtonian impact theory holds for hypersonic continuum flows and is therefore applicable well within the atmosphere, whereas FM theory is applicable for initial atmospheric entry during which the atmosphere is highly rarefied. We consider the case of hypersonic flow, which is typical for atmospheric entry, and, consequently, ignore the thermal velocities of gas molecules. Although entry vehicles are often spin stabilized to enhance their inherent static stability, here we focus on static stability.

To proceed, the axisymmetric shape of the heat shield is represented in spherical polar coordinates by the function $r = f(\theta)$, where $r = 0$ is the center of mass of the vehicle, θ is the zenith angle from the z axis, and $\theta = 0$ corresponds to the forefront point on the heat shield; see Fig. 1. The angle of attack, Φ, is defined as the rotation angle of the vehicle about an axis perpendicular to the z axis and is considered to be small, so that the shadowing of the shield is eliminated. This axis passes through the center of mass and shall henceforth be referred to as the i axis, with corresponding unit vector $\boldsymbol{i}$; see Fig. 1. For small Φ, the torque acting on the vehicle is

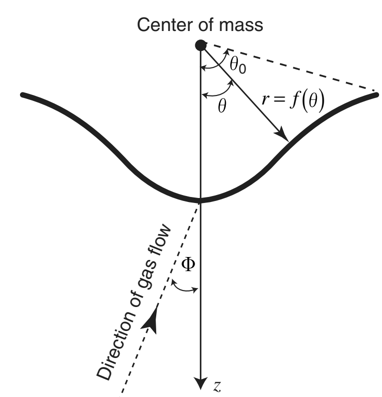

Fig. 1 Schematic of heat shield geometry, showing coordinate system and Φ, which defines the direction of gas flow relative to the frame of reference of the vehicle. The i axis is perpendicular to the page.

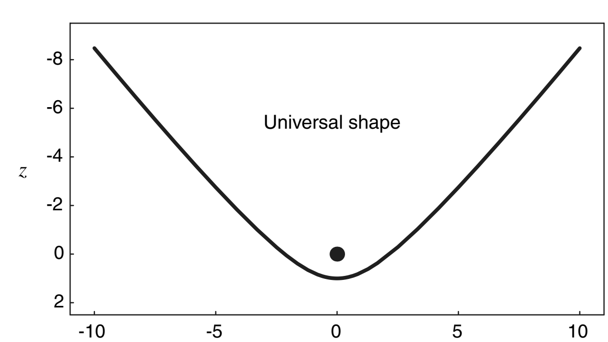

Fig. 2 Universal heat shield shape; the center of mass (dot) is the reference. Axes correspond to a scaled length normalized by the distance of the center of mass to the front of the heat shield.

$$\boldsymbol{\tau} = -\frac{\pi}{2}\rho U^2 \sin(2\Phi)\boldsymbol{i}\begin{cases} \int_0^{\theta_0}[(1-\eta)f^2 \sin\theta(\sin\theta F(\theta) + 2\cos\theta G(\theta)) + \eta \frac{4f^2 f'}{f^2+f'^2}\sin\theta F(\theta)G(\theta)]\,\mathrm{d}\theta, & \text{FM flow} \\ \int_0^{\theta_0} \frac{2f^2 f'}{f^2+f'^2}\sin\theta F(\theta)G(\theta)\,\mathrm{d}\theta, & \text{continuum flow} \end{cases} \tag{1}$$

where ρ is the gas density, U is the speed of the spacecraft, η is the fraction of collisions that result in specular reflection, $F(\theta) = f' \cos\theta - f \sin\theta$, $G(\theta) = f \cos\theta + f' \sin\theta$, and θ_0 defines the extension of the heat shield; see Fig. 1. Note the different expressions for $\boldsymbol{\tau}$ in the FM and continuum regimes. A torque in the positive $\boldsymbol{i}$ direction indicates a stabilizing torque.

Derivation of Heat Shield Shape

We use the calculus of variations [6] to obtain the heat shield shape that yields invariance in torque, that is, a small perturbation to the shape does not change the resulting torque; our solution applies to small angles of attack, as specified earlier. The value of $f(\theta_0)$ is fixed, whereas the front end of the heat shield is required to be flat, that is, $f'(0) = 0$. No other restriction is placed on the form of $f(\theta)$. Application of the Euler–Lagrange equation then yields

$$\begin{aligned} &f^6(\sin\theta - 3\sin 3\theta) + 64ff'^5\sin^2\theta\cos\theta + f'^6(5\sin\theta - 3\sin 3\theta) \\ &- 4f^5 f'(\cos\theta - 3\cos 3\theta) + 2f^2 f'^3 \sin\theta[f'(5 + 27\cos 2\theta) \\ &- 4f''\sin 2\theta] + 8f^5 f''\cos 2\theta\sin\theta + 2f^4 f'\sin\theta[f'(3 + 11\cos 2\theta) \\ &+ 12f''\sin 2\theta] + 4f^3 f'^2[f'(3\cos 3\theta - \cos\theta) \\ &+ 3f''(\sin\theta - \sin 3\theta)] = 0 \end{aligned} \tag{2}$$

for both continuum and FM flows. The resulting differential equation is sufficiently complex that numerical methods are required for its solution.¶ All distances are subsequently normalized with respect to the distance between the center of mass of the craft and the front of the heat shield, that is, $f(0) = 1$. This yields the unique curve in Fig. 2 that exhibits invariance in torque for axisymmetric shape perturbations about the z axis. It is convenient to describe a cross section of the heat shield through the origin in Cartesian coordinates; x is defined as the perpendicular distance from the z axis to the surface in Fig. 2.

Although this solution is derived assuming axisymmetric perturbations to the heat shield shape, it also applies to nonaxisymmetric shapes that possess a circular rim at $\theta = \theta_0$. Such nonaxisymmetric shapes would yield three Cartesian components for the torque, in general. Importantly, the component that dictates the static stability of the vehicle is in the i direction (defined earlier). Implementation of the calculus of variations to this torque component, for such nonaxisymmetric geometries, would yield a single partial differential equation for the heat shield shape, for which the independent variables are the zenith angle θ from the z axis and the azimuthal angle ϕ about the z axis. The boundary conditions for the shape at $\theta = 0$ and θ_0 would remain unchanged from the axisymmetric problem. As such, a solution to this governing equation is the axisymmetric shape in Fig. 2, because this would eliminate the ϕ dependence from the partial differential equation and Eq. (2) would be recovered. This immediately establishes that the derived shape yields torque invariance for both axisymmetric and nonaxisymmetric shape perturbations about the z axis.

Interestingly, the unique shape in Fig. 2 is identical in both the continuum and FM regimes (for arbitrary thermal accommodation), despite the dramatically different torques experienced, and closely resembles the sphere-cone geometry commonly used. It is important to note that this generic shape arises naturally from the requirement of invariance in torque and is not explicitly specified in the formulation. This differs from previous variational solutions, in which the generic shape under investigation was explicitly stated. For example, Bowman and Lewis [7] evaluated the power-law shape that produced minimum drag in rarefied flows, whereas Bunimovich and Dubinskii [8] calculated the surface of revolution for a specified length and diameter that gives minimum drag for varying degrees of rarefaction. These are in contrast to the present work, in which no restriction is placed on the geometry of the body and invariance in torque, rather than drag, is studied.

¶Mathematica 6.0 available from Wolfram Research, Inc.

Geometry of the Heat Shield

To investigate the form of the heat shield, in Fig. 3 we present results for its curvature as a function of x, which is the distance from the symmetry axis. Note the distinct property of constant curvature at the front of the heat shield for normalized distances of less than 1 from the symmetry axis; lengths are normalized by the distance from the center of mass to the front of the heat shield. This immediately establishes that the front of the derived heat shield is indeed a spherical cap.

This front curvature drops rapidly to zero for normalized distances greater than 1 from the axis, that is, a conical section. Importantly, at large normalized distances from the axis, the minor angle of the cone is found to be $\arcsin(1/\sqrt{3}) \approx 35.26$ deg, that is, $\theta_0 \approx 144.74$ deg (Fig. 1). This shows that the widely used spherically capped cone, which has been developed using experimental design and computational simulation, can be derived mathematically.

The derived shape in Fig. 2 is well described in Cartesian coordinates by the (approximate) empirical formula

$$\begin{aligned} \tilde{f}(x) = 1 - (2/\pi^2)[&\sqrt{2}\pi x \arctan(\pi x/(4\sqrt{2})) \\ &- 4\log(1 + (\pi^2 x^2/32))] \end{aligned} \tag{3}$$

which is exact in the asymptotic limit $x \to 0$ (front of heat shield). The relative error of this approximation compared with the solution to Eq. (2), defined as $(\tilde{f} - 1)/(f - 1)$, tends to its maximum of 4% as $x \to \infty$ (conical section of heat shield). A comparison of this approximate formula to the exact numerical solution for the curvature and shape is given in Fig. 3, in which good agreement is found. Also shown is the (empirically derived) hyperbola

$$(z - 5)^2 - 2x^2 = 16 \tag{4}$$

Although a less accurate approximation (maximum error of 12% as $x \to \infty$, exact as $x \to 0$) than Eq. (3), Eq. (4) gives a more transparent geometric interpretation.

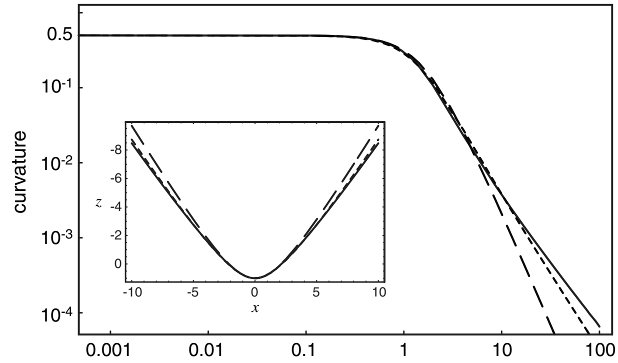

Fig. 3 Normalized curvature of the derived heat shield as a function of the normalized distance from its symmetry axis. The normalized radius of curvature at the front is 2; solid line: derived solution, dotted line: approximation in Eq. (3), dashed line: hyperbola in Eq. (4). Inset: direct comparison of the heat shield shapes.

Proof of Maximum Torque

A key question is whether the derived solution gives the minimum or maximum torque. A maximum stabilizing torque requires the integrals in Eq. (1) to achieve their minimum value; note that these integrals are multiplied by a negative quantity in Eq. (1). To determine the nature of the derived extremum, we introduce axisymmetric perturbations of type $\epsilon g(\theta)$ to the solution $f(\theta)$, where $\epsilon \ll 1$ and the only restriction on $g(\theta)$ is that it must vanish at $\theta = \theta_0$, that is, the endpoint is unchanged by the perturbation. The torque is calculated by replacing $f(\theta)$ with $f(\theta) + \epsilon g(\theta)$ in Eq. (1). Each of the integrals can be represented by its Taylor series expansion: $I(\epsilon) = I(0) + I'(0)\epsilon + I''(0)\epsilon^2/2 + O(\epsilon^3)$. In this expression, $I(0)$ is the value of the integral using the derived solution $f(\theta)$, and $I'(0) = 0$ for all acceptable functions $g(\theta)$, because this is an equivalent statement to the Euler–Lagrange equation. If $I''(0) > 0$ for all $g(\theta)$, the function $f(\theta)$ gives a weak minimum [9] for I and a maximum stabilizing torque. To ensure $I''(0) > 0$, two conditions must be satisfied [9].

Condition 1: This condition is identical for the continuum and FM regimes and requires

$$\sin\theta[f\cos(2\theta)(f^2 - 3f'^2) + f'\sin(2\theta)(3f^2 - f'^2)] > 0 \quad (5)$$

for $0 < \theta < \theta_0$. The inequality in Eq. (5) is satisfied for all $\theta_0 < \arcsin(1/\sqrt{3})$, that is, all extensions of the heat shield.

Condition 2: It is also necessary that there exists a nonvanishing function, $u(\theta)$, that satisfies

$$Ru'' + R'u' + (Q' - P)u = 0 \quad (6)$$

where P, Q, and R are functions of θ that depend on the shape of the heat shield. Although these functions are different for the continuum and FM regimes, the combination in which they appear in Eq. (6) gives independence of the accommodation coefficient. Conveniently, the FM and continuum cases collapse onto each other, establishing that the same function $u(\theta)$ will satisfy Eq. (6) for both the FM and continuum regimes. Solution with, for example, $u(0) = u'(0) = 1$ gives a nonvanishing solution.

As the required conditions are met for both the integrals in Eq. (1), each attains a weak minimum with $f(\theta)$, implying a maximum torque. Perturbations that can not be written as $\epsilon g(\theta)$ have not been considered in this analysis; our solution is not a strong minimum [9]. Importantly, the derived solution gives the maximum stabilizing torque in both the continuum and FM regimes for all extensions of the heat shield θ_0. Extension of this proof to nonaxisymmetric shape perturbations presents a more difficult challenge and is an outstanding problem.

Stability

For continuum flows, the obtained shape is always stable, regardless of θ_0; see Fig. 1. For FM flows, stability requires θ_0 exceed a critical value, shown as the solid line in Fig. 4; this solid line is well approximated by $\theta_0 = \arccos([\eta^3 + \eta]/2)$. For θ_0 greater than this value, the stabilizing torque increases rapidly with increasing θ_0; $\theta_0 > 90$ deg always yield stability.

To illustrate the utility of the derived universal shape, we present two nominal entry vehicles with different centers of mass in Fig. 5, the heat shield of which is immediately apparent from Fig. 2. These vehicles possess a striking resemblance to current entry vehicles. Both vehicles in Fig. 5 are statically stable in the continuum regime, whereas only the top vehicle is guaranteed stability in the FM regime. Stability of the lower vehicle in the FM regime depends on the thermal accommodation of its surface, because θ_0 is less than 90 deg; see Fig. 4. These examples illustrate that static stability may not be possible for some vehicle configurations, because the variational solution gives the shape that yields maximum torque, and this is negative (unstable) in some cases.

Importantly, the design of practical heat shields involves numerous competing factors [10], which include the expected heat load and the craft volumetric efficiency, in addition to aerodynamic stability. We thus emphasize that the presented results focus on only one component of this multi-objective problem.

The universality of the derived shape for FM and continuum flows suggests that it applies under more general conditions. It remains to be seen whether this shape produces similar invariance for flows in the transition regime and when thermochemical effects are included [1]. This would necessitate the use of computational methods, which would eliminate the ability for mathematical analysis.

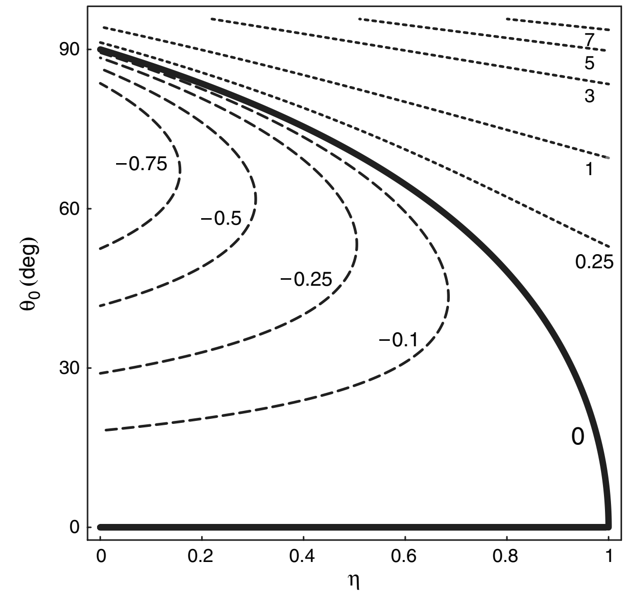

Fig. 4 Stabilizing torque as function of θ_0 and specular reflection coefficient η. A negative value indicates a destabilizing torque. Normalized torque values indicated (scaling: $\pi\rho U^2 L^3 \sin(2\Phi)/2$, where L is the distance from the center of mass to the front of the heat shield).

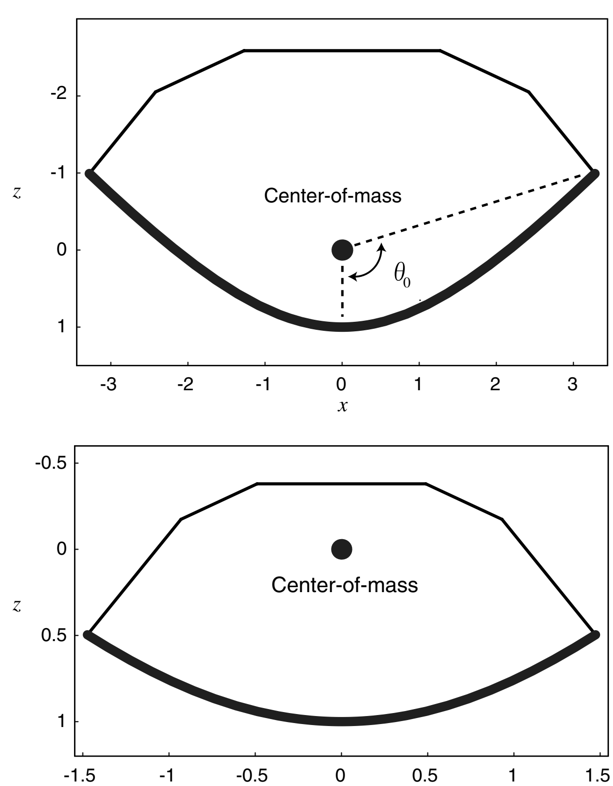

Fig. 5 Sample entry vehicles constructed using curve in Fig. 2 showing the center of mass (dot) and θ_0.

Conclusions

In summary, we have shown that the generic shape of the commonly used blunted-cone can be derived and yields a mathematically optimal solution with respect to torque invariance. The result is a single universal shape that depends only on the center of mass of the vehicle and is applicable in both the FM and continuum regimes, yielding maximum torque and, hence, maximum static stability.

Acknowledgment

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